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# **$\tau$ - and $\mu$ -physics in a general two Higgs doublet model with $\mu - \tau$ flavor violation**

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## Abstract

Motivated by the recent CMS excess in a flavor violating Higgs decay  $h \rightarrow \mu\tau$  as well as the anomaly of muon anomalous magnetic moment (muon g-2), we consider a scenario where both the excess in  $h \rightarrow \mu\tau$  and the anomaly of muon g-2 are explained by the  $\mu - \tau$  flavor violation in a general two Higgs doublet model. We study various processes involving  $\mu$  and  $\tau$ , and then discuss the typical predictions and constraints in this scenario. Especially, we find that the prediction of  $\tau \rightarrow \mu\gamma$  can be within the reach of the Belle II experiment. We also show that the lepton non-universality between  $\tau \rightarrow \mu\nu\bar{\nu}$  and  $\tau \rightarrow e\nu\bar{\nu}$  can be sizable, and hence the analysis of the current Belle data and the future experimental improvement would have an impact on this model. Besides, processes such as  $\tau \rightarrow \mu l^+ l^-$  ( $l = e, \mu$ ),  $\tau \rightarrow \mu\eta$ ,  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$ , and muon EDM can be accessible, depending on the unknown Yukawa couplings. On the other hand, the processes like  $\tau \rightarrow e\gamma$  and  $\tau \rightarrow e l^+ l^-$  ( $l = e, \mu$ ) could not be sizable to observe because of the current strong constraints on the  $e - \mu$  and  $e - \tau$  flavor violations. Then we also conclude that contrary to  $h \rightarrow \mu\tau$  decay mode, the lepton flavor violating Higgs boson decay modes  $h \rightarrow e\mu$  and  $h \rightarrow e\tau$  are strongly suppressed, and hence it will be difficult to observe these modes at the LHC experiment.

## I. INTRODUCTION

The Standard Model (SM) of the elementary particles describes particle physics phenomena remarkably well up to the electroweak scale. In addition, the recent discovery of a Higgs boson at the LHC [1, 2] strengthens the success of the SM. On the other hand, the detailed measurements of the Higgs boson properties have just started, and the whole structure of the Higgs sector may have not been unveiled. Therefore, theoretical and experimental studies of the extended Higgs sector would be important to understand the nature of the Higgs sector.

One of simple extensions of the Higgs sector in the SM is a two Higgs doublet model (2HDM) where additional Higgs doublet is introduced and both Higgs doublets can couple to all fermions. As a result, flavor violating phenomena mediated by the Higgs bosons are predicted [3]. In most cases, such a flavor violation has been considered to be avoided because of lack of the experimental supports for the anomalous flavor-violating phenomena [4–7].

However, the CMS collaboration has recently reported an event excess in a flavor-violating Higgs decay  $h \rightarrow \mu\tau$  [8], and it suggests that the best fit value of the branching ratio is

$$\text{BR}(h \rightarrow \mu\tau) = (0.84^{+0.39}_{-0.37}) \%, \quad (1)$$

where the final state is a sum of  $\mu^+\tau^-$  and  $\mu^-\tau^+$ , and the deviation from the SM prediction is  $2.4\sigma$ . In addition, the result of the ATLAS experiment has also appeared recently [9], and it is shown as

$$\text{BR}(h \rightarrow \mu\tau) = (0.77 \pm 0.62) \%, \quad (2)$$

which is consistent with the CMS result within  $1\sigma$ . Although these results have not been conclusive yet, these become strong motivations to study the flavor violating phenomena predicted by the Beyond Standard Models [10–32].<sup>1</sup>

In Ref. [14], we pointed out a possibility that the  $\mu - \tau$  flavor violation in general 2HDM can explain not only the CMS excess in the Higgs decay  $h \rightarrow \mu\tau$ , but also

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<sup>1</sup> The lepton flavor violating Higgs decays have been investigated before the CMS excess has been reported[33–39].

the anomaly of muon anomalous magnetic moment (muon g-2) [40]. This possibility is interesting because two unexplained phenomena can be accommodated in the 2HDM, and hence it is worth further investigating this possibility. In this paper, we study phenomena related to  $\mu$  and  $\tau$  lepton physics in the scenario to see whether there are any interesting predictions and constraints caused by the  $\mu - \tau$  flavor violation.

The paper is organized as follows. In section II, we present a general 2HDM where both Higgs doublets couple to all fermions. We discuss the Yukawa interactions and Higgs mass spectrum in the model. In section III, we consider a solution where the CMS excess in  $h \rightarrow \mu\tau$  decay as well as muon g-2 anomaly can be explained by the  $\mu - \tau$  flavor violating Yukawa interactions in the model. We show the typical parameter space where both anomalies are explained. In section IV, we discuss  $\tau$ - and  $\mu$ -physics in the interesting region studied in the previous section. Especially, we study  $\tau \rightarrow \mu\gamma$ ,  $\mu \rightarrow e\gamma$ , muon electric dipole moment (muon EDM),  $\tau \rightarrow \mu\nu\bar{\nu}$ ,  $\tau^- \rightarrow \mu^- l^+ l^-$  ( $l = e, \mu$ ),  $\mu^+ \rightarrow e^+ e^- e^+$ , and  $\tau \rightarrow \mu\eta$ . The prediction of  $\tau \rightarrow \mu\gamma$  can be within the reach of the Belle II experiment, which will start near future. The extra Higgs boson correction to  $\tau \rightarrow \mu\nu\bar{\nu}$  can be as large as  $10^{-3} - 10^{-4}$ , but it is not large in  $\tau \rightarrow e\nu\bar{\nu}$  mode. The future improvement of measurement on lepton flavor universality in  $\tau \rightarrow \mu(e)\nu\bar{\nu}$  decay will be important in the scenario. We also find that unlike the  $\mu - \tau$  flavor violation suggested by the CMS result, the  $e - \tau$  and  $e - \mu$  flavor violations are severely constrained since the constraint from the  $\mu \rightarrow e\gamma$  is strong. Therefore, the processes related to  $e - \tau$  and  $e - \mu$  flavor violations are suppressed. In section V, we also discuss an implication to Higgs physics. Since  $e - \tau$  and  $e - \mu$  flavor violating Yukawa couplings are strongly suppressed,  $h \rightarrow e\tau$  and  $h \rightarrow e\mu$  Higgs decay modes will not be observed in this scenario, contrary to  $h \rightarrow \mu\tau$  mode. In section VI, we summarize our results.

## II. GENERAL TWO HIGGS DOUBLET MODEL

In a general two Higgs doublet model, there are no symmetry to distinguish the two different Higgs doublets. Thus, both the Higgs doublets can couple to all fermions, and hence there are flavor violating interactions in Higgs sector. In

general, when the Higgs potential is minimized in the SM-like vacuum, both neutral components of Higgs doublets develop nonzero vacuum expectation values (vevs). Taking a certain linear combination, we can define the basis where only one of the Higgs doublets obtains the nonzero vev as follows:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+\phi_1+iG}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+iA}{\sqrt{2}} \end{pmatrix}, \quad (3)$$

where  $G^+$  and  $G$  are Nambu-Goldstone bosons, and  $H^+$  and  $A$  are a charged Higgs boson and a CP-odd Higgs boson, respectively. Then  $H^+$  and  $A$  are in the mass eigenstates. The CP-even neutral Higgs bosons  $\phi_1$  and  $\phi_2$  can mix and form mass eigenstates,  $h$  and  $H$  ( $m_H > m_h$ ), in general:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\beta\alpha} & \sin \theta_{\beta\alpha} \\ -\sin \theta_{\beta\alpha} & \cos \theta_{\beta\alpha} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}. \quad (4)$$

Here  $\theta_{\beta\alpha}$  is the mixing angle and fixed by the Higgs potential analysis. Note that when  $\cos \theta_{\beta\alpha} \rightarrow 0$  ( $\sin \theta_{\beta\alpha} \rightarrow 1$ ), the interactions of  $\phi_1$  approach to those of the SM Higgs boson.

### A. Yukawa interactions

In mass eigenbasis for the fermions, the Yukawa interactions are expressed by

$$\begin{aligned} \mathcal{L} = & -\bar{Q}_L^i H_1 y_d^i d_R^i - \bar{Q}_L^i H_2 \rho_d^{ij} d_R^j - \bar{Q}_L^i (V_{\text{CKM}}^\dagger)^{ij} \tilde{H}_1 y_u^j u_R^j - \bar{Q}_L^i (V_{\text{CKM}}^\dagger)^{ij} \tilde{H}_2 \rho_u^{jk} u_R^k \\ & -\bar{L}_L^i H_1 y_e^i e_R^i - \bar{L}_L^i H_2 \rho_e^{ij} e_R^j, \end{aligned} \quad (5)$$

where  $i$  and  $j$  represent flavor indices, and  $Q = (V_{\text{CKM}}^\dagger u_L, d_L)^T$ ,  $L = (V_{\text{MNS}} \nu_L, e_L)^T$  are defined.  $V_{\text{CKM}}$  is the Cabbibo-Kobayashi-Maskawa (CKM) matrix and  $V_{\text{MNS}}$  is the Maki-Nakagawa-Sakata (MNS) matrix. Fermions ( $f_L$ ,  $f_R$ ) ( $f = u, d, e, \nu$ ) are mass eigenstates. Here we have assumed that the tiny neutrino masses are achieved by the seesaw mechanism introducing super-heavy right-handed neutrinos, so that in the low-energy effective theory, the left-handed neutrinos have a  $3 \times 3$  Majorana mass matrix. The Yukawa coupling constants  $\rho_f^{ij}$  are general  $3 \times 3$  complex matrices and can be sources of the Higgs-mediated flavor changing processes.

In mass eigenstates of Higgs bosons, the Yukawa interactions are given by

$$\begin{aligned}\mathcal{L} = & - \sum_{f=u,d,e} \sum_{\phi=h,H,A} y_{\phi ij}^f \bar{f}_{Li} \phi f_{Rj} + \text{h.c.} \\ & - \bar{\nu}_{Li} (V_{\text{MNS}}^\dagger \rho_e)^{ij} H^+ e_{Rj} - \bar{u}_i (V_{\text{CKM}} \rho_d P_R - \rho_u^\dagger V_{\text{CKM}} P_L)^{ij} H^+ d_j + \text{h.c.},\end{aligned}\quad (6)$$

where

$$\begin{aligned}y_{h\ ij}^f &= \frac{m_f^i}{v} s_{\beta\alpha} \delta_{ij} + \frac{\rho_f^{ij}}{\sqrt{2}} c_{\beta\alpha}, \\ y_{H\ ij}^f &= \frac{m_f^i}{v} c_{\beta\alpha} \delta_{ij} - \frac{\rho_f^{ij}}{\sqrt{2}} s_{\beta\alpha}, \\ y_{A\ ij}^f &= \begin{cases} -\frac{i\rho_f^{ij}}{\sqrt{2}} & (\text{for } f = u), \\ \frac{i\rho_f^{ij}}{\sqrt{2}} & (\text{for } f = d, e) \end{cases}\end{aligned}\quad (7)$$

are defined with  $c_{\beta\alpha} \equiv \cos \theta_{\beta\alpha}$  and  $s_{\beta\alpha} \equiv \sin \theta_{\beta\alpha}$ . Note that when  $c_{\beta\alpha}$  is small, the Yukawa interactions of  $h$  are almost equal to those of the SM Higgs boson, however, there are small flavor violating interactions  $\rho_f^{ij}$  which are suppressed by  $c_{\beta\alpha}$ . On the other hand, the Yukawa interactions of heavy Higgs bosons ( $H$ ,  $A$ , and  $H^+$ ) mainly come from the  $\rho_f^{ij}$  couplings.

## B. Higgs mass spectrum

Let us comment on the relation between the Higgs masses and the parameters in the Higgs potential. The renormalizable Higgs potential in the general 2HDM is given by

$$\begin{aligned}V = & M_{11}^2 H_1^\dagger H_1 + M_{22}^2 H_2^\dagger H_2 - \left( M_{12}^2 H_1^\dagger H_2 + \text{h.c.} \right) \\ & + \frac{\lambda_1}{2} (H_1^\dagger H_1)^2 + \frac{\lambda_2}{2} (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1) (H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2) (H_2^\dagger H_1) \\ & + \frac{\lambda_5}{2} (H_1^\dagger H_2)^2 + \left\{ \lambda_6 (H_1^\dagger H_1) + \lambda_7 (H_2^\dagger H_2) \right\} (H_1^\dagger H_2) + \text{h.c.}.\end{aligned}\quad (8)$$

In the basis shown in Eq. (3), the Higgs boson masses can be described by the dimensionless parameters and  $M_{22}$  using the stationary conditions for the Higgs

doublets:

$$\begin{aligned}
m_{H^+}^2 &= M_{22}^2 + \frac{v^2}{2}\lambda_3, \\
m_A^2 - m_{H^+}^2 &= -\frac{v^2}{2}(\lambda_5 - \lambda_4), \\
(m_H^2 - m_h^2)^2 &= \{m_A^2 + (\lambda_5 - \lambda_1)v^2\}^2 + 4\lambda_6^2v^4, \\
\sin 2\theta_{\beta\alpha} &= -\frac{2\lambda_6v^2}{m_H^2 - m_h^2}.
\end{aligned} \tag{9}$$

Especially, when  $c_{\beta\alpha}$  is close to zero (that is,  $\lambda_6 \sim 0$ ), we obtain the following simple expressions for the Higgs boson masses:

$$\begin{aligned}
m_h^2 &\simeq \lambda_1v^2, \\
m_H^2 &\simeq m_A^2 + \lambda_5v^2, \\
m_{H^+}^2 &= m_A^2 - \frac{\lambda_4 - \lambda_5}{2}v^2, \\
m_A^2 &= M_{22}^2 + \frac{\lambda_3 + \lambda_4 - \lambda_5}{2}v^2.
\end{aligned} \tag{10}$$

Note that fixing the couplings,  $\lambda_3$ ,  $\lambda_4$  and  $\lambda_5$ , the heavy Higgs boson masses are expressed by the CP-odd Higgs boson mass  $m_A$ , which we treat as a free parameter of the model. The contribution to the Peskin-Takeuchi T-parameter [41] should be taken into account, so that we assume that it is suppressed by the degeneracy between  $m_A$  and  $m_{H^+}$  as well as the small Higgs mixing parameter  $c_{\beta\alpha}$ . Therefore, we set  $\lambda_4 = \lambda_5$  in our analysis, which corresponds to  $m_A = m_{H^+}$ .

### III. SOLUTION TO THE CMS EXCESS IN $h \rightarrow \mu\tau$ AND THE MUON G-2 ANOMALY

The CMS collaboration has reported an excess in a Higgs boson decay mode  $h \rightarrow \mu\tau$ . Furthermore, it is known that there is a discrepancy between the measured value and the SM prediction of the muon anomalous magnetic moment (muon g-2). The both anomalies cannot be explained by the SM, and hence they might be an indication of physics beyond the SM. In this section, we discuss whether the general 2HDM can accommodate both anomalies simultaneously, and investigate the parameter space where both anomalies can be explained.

### A. $h \rightarrow \mu\tau$

An excess in  $h \rightarrow \mu\tau$  decay mode has been reported by the CMS collaboration: The best fit value of the branching ratio is  $\text{BR}(h \rightarrow \mu\tau) = (0.84^{+0.39}_{-0.37})\%$  [8]. As discussed in the Introduction, the ATLAS collaboration has also shown the result,  $\text{BR}(h \rightarrow \mu\tau) = (0.77 \pm 0.62)\%$  [9], which is consistent with the CMS one. It would be an indication of new physics because the SM can not accommodate the excess. Since in the general 2HDM the SM-like Higgs boson has flavor violating Yukawa interactions as discussed in the previous section, the excess can be easily explained. The expression of the branching ratio of the  $h \rightarrow \mu\tau$  process is given by

$$\begin{aligned} \text{BR}(h \rightarrow \mu\tau) &= \frac{\Gamma(h \rightarrow \mu^+\tau^-) + \Gamma(h \rightarrow \mu^-\tau^+)}{\Gamma_h} \\ &= \frac{c_{\beta\alpha}^2(|\rho_e^{\mu\tau}|^2 + |\rho_e^{\tau\mu}|^2)m_h}{16\pi\Gamma_h}, \end{aligned} \quad (11)$$

where  $\Gamma_h$  is a total decay width of Higgs boson  $h$  and we adopt  $\Gamma_h = 4.1$  MeV in this paper. In order to accommodate the CMS excess, the  $\mu - \tau$  flavor violating Yukawa couplings need to satisfy the following condition:

$$\begin{aligned} \bar{\rho}^{\mu\tau} &\equiv \sqrt{\frac{|\rho_e^{\mu\tau}|^2 + |\rho_e^{\tau\mu}|^2}{2}} \\ &\simeq 0.26 \left( \frac{0.01}{|c_{\beta\alpha}|} \right) \sqrt{\frac{\text{BR}(h \rightarrow \mu\tau)}{0.84 \times 10^{-2}}}. \end{aligned} \quad (12)$$

It is interesting to note that even in the small Higgs mixing ( $|c_{\beta\alpha}| \simeq 0.01$ ), the  $\mu - \tau$  flavor violating Yukawa couplings with the order of 0.1 can achieve the CMS excess.

### B. The muon anomalous magnetic moment (muon g-2)

We have shown that the  $\mu - \tau$  flavor violating Yukawa couplings in the general 2HDM explain the CMS excess in the  $h \rightarrow \mu\tau$  decay mode. Here we consider extra contributions to the muon anomalous magnetic moment (muon g-2) generated by the  $\mu - \tau$  flavor violating Yukawa couplings.

A discrepancy between the measured value ( $a_\mu^{\text{Exp}}$ ) and the standard model prediction ( $a_\mu^{\text{SM}}$ ) of the muon g-2 has been reported [40]. For example, Ref. [42] shows the following result:

$$a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10}. \quad (13)$$

Here we consider whether the extra contributions induced by the  $\mu-\tau$  flavor violating interactions can accommodate the muon g-2 anomaly. The effective operator for the muon g-2 is expressed by

$$\mathcal{L} = \frac{v}{\Lambda^2} \bar{\mu}_L \sigma^{\mu\nu} \mu_R F_{\mu\nu} + \text{h.c.} \quad (14)$$

We note that the chirality of muon is flipped in the operator. Therefore, if there is a large chirality flip induced by the new physics, it can enhance the extra contributions to the muon g-2 [43]. A Feynman diagram for the extra contributions of the neutral Higgs bosons to the muon g-2, induced by the  $\mu-\tau$  flavor violating Yukawa couplings, is shown in Fig. 1. As shown in Fig. 1, the chirality flip occurs in the internal line

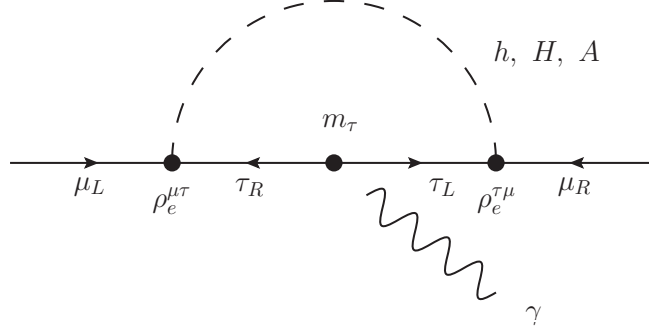


FIG. 1: A Feynman diagram for neutral Higgs boson contributions to the muon g-2. A photon is attached somewhere in the charged lepton line.

of  $\tau$  lepton in the diagram. Therefore, it induces the  $O(m_\tau/m_\mu)$  enhancement in the extra contributions to the muon g-2, compared with the one generated by the flavor-diagonal Yukawa coupling. We stress that the  $\mu-\tau$  flavor violating interaction is essential to obtain such an enhancement. Note that both couplings  $\rho_e^{\mu\tau}$  and  $\rho_e^{\tau\mu}$  should be nonzero to get the chirality flip in the internal  $\tau$  lepton line. The expression of the enhanced extra contribution  $\delta a_\mu$  is given by

$$\delta a_\mu = \frac{m_\mu m_\tau \rho_e^{\mu\tau} \rho_e^{\tau\mu}}{16\pi^2} \left[ \frac{c_{\beta\alpha}^2 (\log \frac{m_h^2}{m_\tau^2} - \frac{3}{2})}{m_h^2} + \frac{s_{\beta\alpha}^2 (\log \frac{m_H^2}{m_\tau^2} - \frac{3}{2})}{m_H^2} - \frac{\log \frac{m_A^2}{m_\tau^2} - \frac{3}{2}}{m_A^2} \right], \quad (15)$$

where we have assumed that  $\rho_e^{\mu\tau} \rho_e^{\tau\mu}$  is real, for simplicity. We will discuss the effect of an imaginary part of these Yukawa couplings later. We note that a degeneracy of



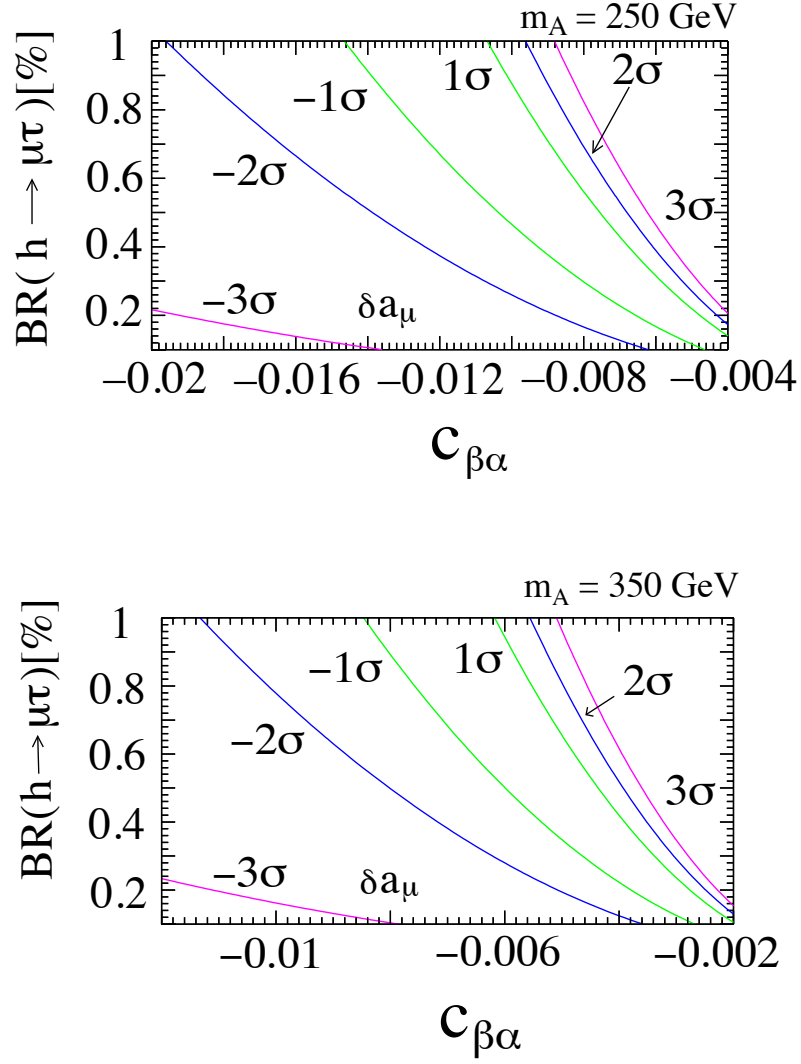


FIG. 2: Numerical result for  $\delta a_\mu$  as a function of  $c_{\beta\alpha}$  and  $\text{BR}(h \rightarrow \mu\tau)$  for  $m_A = 250$  GeV (upper figure) and 350 GeV (lower figure). Regions where the muon g-2 anomaly in Eq. (13) is explained within  $\pm 1\sigma$ ,  $\pm 2\sigma$  and  $\pm 3\sigma$  are shown. Here we determine the mass spectrum of heavy Higgs bosons assuming  $\lambda_4 = \lambda_5 = 0.5$  in Eq. (10). We have assumed  $\rho_e^{\mu\tau} \rho_e^{\tau\mu} < 0$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$  to obtain the positive contribution to  $\delta a_\mu$ .

all neutral Higgs bosons suppresses the extra contribution to the muon g-2, as seen in Eq. (15).

In Fig. 2, we show numerical results for the extra contribution to muon g-2 ( $\delta a_\mu$ ) as a function of  $c_{\beta\alpha}$  and  $\text{BR}(h \rightarrow \mu\tau)$  for  $m_A = 250$  GeV (upper figure) and 350

GeV (lower figure). Regions where the muon g-2 anomaly in Eq. (13) is explained within  $\pm 1\sigma$ ,  $\pm 2\sigma$  and  $\pm 3\sigma$  are shown. Here we have fixed the mass spectrum of heavy Higgs bosons assuming  $\lambda_4 = \lambda_5 = 0.5$  in Eq. (10). We have assumed that  $\rho_e^{\mu\tau}\rho_e^{\tau\mu} < 0$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$  to obtain the positive contribution to  $\delta a_\mu$ . We only discuss the case with  $c_{\beta\alpha} < 0$ , however, the predictions of  $\delta a_\mu$  and  $\text{BR}(h \rightarrow \mu\tau)$  do not change even if the sign of  $c_{\beta\alpha}$  is flipped ( $c_{\beta\alpha} \rightarrow -c_{\beta\alpha}$ ). One can see that there are regions where both anomalies of the muon g-2 and  $h \rightarrow \mu\tau$  can be explained in the 2HDM. We note that although larger  $\text{BR}(h \rightarrow \mu\tau)$  is preferred by the muon g-2 anomaly, the regions where  $\text{BR}(h \rightarrow \mu\tau)$  is smaller than the one suggested by the CMS result are also allowed by the muon g-2 anomaly if  $|c_{\beta\alpha}|$  is small.

In Fig. 3, the numerical result for the  $\delta a_\mu$  is shown as a function of  $m_A$  and  $c_{\beta\alpha}$  fixing that  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$ . Regions that explain the muon g-2 anomaly in Eq. (13) within  $\pm 1\sigma$ ,  $\pm 2\sigma$  and  $\pm 3\sigma$  are shown. Here we take  $\lambda_4 = \lambda_5 = 0.5$  to determine the mass spectrum of heavy Higgs bosons (shown in Eq. (10)) as a function of  $m_A$ . We assume that the Yukawa couplings  $\rho_e^{\tau\mu}$  and  $\rho_e^{\mu\tau}$  are determined to realize  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ . When  $|c_{\beta\alpha}|$  gets smaller,  $\delta a_\mu$  increases because the Yukawa couplings  $\rho_e^{\mu\tau(\tau\mu)}$  become larger with the fixed  $\text{BR}(h \rightarrow \mu\tau)$ . It is interesting to see that the 2HDM can explain both anomalies of the muon g-2 and  $h \rightarrow \mu\tau$  when  $|c_{\beta\alpha}|$  is small ( $|c_{\beta\alpha}| \sim 0.01$ ) for  $m_A = 200 - 500$  GeV. We note that the small mixing  $|c_{\beta\alpha}|$  is consistent with the current results of the Higgs coupling measurements as well as the constraints from the electroweak observables.

In Fig. 4, similar to Fig. 3, the numerical result for the  $\delta a_\mu$  is shown as a function of  $m_A$  and  $\lambda_5$  fixing that  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  and  $c_{\beta\alpha} = -0.008$ . We have assumed that  $\rho_e^{\tau\mu} = -\rho_e^{\mu\tau}$  and  $\lambda_4 = \lambda_5$ . As  $\lambda_5$  gets larger, the  $\delta a_\mu$  becomes larger because the non-degeneracy between  $H$  and  $A$  increases and it enhances the  $\delta a_\mu$ . Figs. 2, 3 and 4 show the typical interesting regions which explain both anomalies of the muon g-2 and  $h \rightarrow \mu\tau$ .

#### IV. $\tau$ - AND $\mu$ -PHYSICS IN THIS SCENARIO

So far, we have seen that the general 2HDM with the  $\mu - \tau$  flavor violation accommodates the muon g-2 anomaly and the CMS excess in  $h \rightarrow \mu\tau$  decay, si-

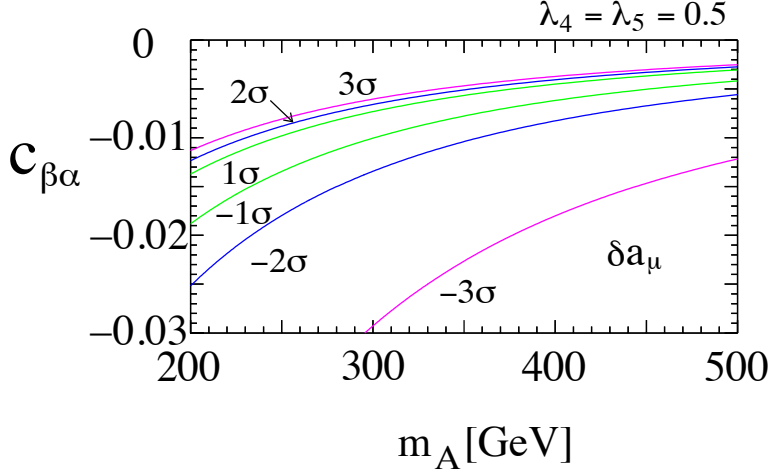


FIG. 3: Numerical result for  $\delta a_\mu$  as a function of  $m_A$  and  $c_{\beta\alpha}$  assuming  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$ . Regions which explain the muon g-2 anomaly in Eq. (13) within  $\pm 1\sigma$ ,  $\pm 2\sigma$  and  $\pm 3\sigma$  are shown. Here we determine the mass spectrum of heavy Higgs bosons as a function of  $m_A$  assuming  $\lambda_4 = \lambda_5 = 0.5$  in Eq. (10). We have assumed  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$  to obtain the positive contribution to  $\delta a_\mu$  and the Yukawa couplings  $\rho_e^{\mu\tau(\tau\mu)}$  are fixed to realize  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$ .

multaneously. The parameter regions with  $|c_{\beta\alpha}| \sim 0.01$  and  $m_A \sim O(100)$  GeV are typically interesting. In this section, we investigate what kinds of predictions and/or constraints in  $\tau$ - and  $\mu$ -physics we have in this scenario.

#### A. $\tau \rightarrow \mu\gamma$

The first process we would like to discuss is  $\tau \rightarrow \mu\gamma$ . The  $\mu - \tau$  flavor violating Yukawa couplings induce the flavor violating phenomena  $\tau \rightarrow \mu\gamma$ , as shown, for example, in Fig. 5. We parametrize the decay amplitude ( $T_{\tau \rightarrow \mu\gamma}$ ) as follows:

$$T_{\tau \rightarrow \mu\gamma} = e\epsilon^{\alpha*}(q)\bar{u}_\mu(p-q)m_\tau i\sigma_{\alpha\beta}q^\beta(A_L P_L + A_R P_R)u_\tau(p), \quad (16)$$

where  $P_{R,L} (= (1 \pm \gamma_5)/2)$  are chirality projection operators, and  $e$ ,  $\epsilon^\alpha$ ,  $q$ ,  $p$  and  $u_f$  are the electric charge, a photon polarization vector, a photon momentum, a  $\tau$  momentum, and a spinor of the fermion  $f$ , respectively. The branching ratio is given

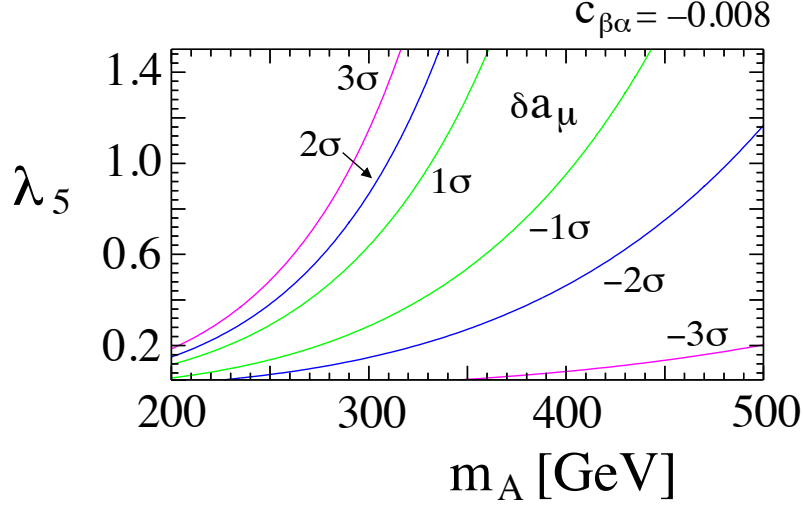


FIG. 4: Numerical result for  $\delta a_\mu$  as a function of  $m_A$  and  $\lambda_5$  assuming  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  with the fixed  $c_{\beta\alpha} (= -0.008)$ . Here we have assumed  $\lambda_4 = \lambda_5$  and  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ .

by

$$\frac{\text{BR}(\tau \rightarrow \mu\gamma)}{\text{BR}(\tau \rightarrow \mu\bar{\nu}_\mu\nu_\tau)} = \frac{48\pi^3\alpha(|A_L|^2 + |A_R|^2)}{G_F^2}, \quad (17)$$

where  $\alpha$  and  $G_F$  are the fine structure constant and the Fermi constant, respectively. The lepton flavor violating Higgs contributions to  $A_L$  and  $A_R$  via  $y_{\phi\tau\tau}^e$  ( $\phi = h, H, A$ ) Yukawa interactions at one-loop level (left figure in Fig. 5) are given by <sup>2</sup>

$$\begin{aligned} A_{L,R} &= \sum_{\phi=h, H, A, H^-} A_{L,R}^\phi, \\ A_L^\phi &= \frac{y_{\phi\tau\mu}^{e*}}{16\pi^2 m_\phi^2} \left[ y_{\phi\tau\tau}^{e*} \left( \log \frac{m_\phi^2}{m_\tau^2} - \frac{3}{2} \right) + \frac{y_{\phi\tau\tau}^e}{6} \right], \quad (\phi = h, H, A) \\ A_R^\phi &= A_L^\phi |_{y_{\phi\tau\mu}^{e*} \rightarrow y_{\phi\mu\tau}^e, y_{\phi\tau\tau}^e \leftrightarrow y_{\phi\tau\tau}^{e*}}, \quad (\phi = h, H, A), \\ A_L^{H^-} &= -\frac{(\rho_e^\dagger \rho_e)^{\mu\tau}}{192\pi^2 m_{H^-}^2}, \quad A_R^{H^-} = 0, \end{aligned} \quad (18)$$

where  $A_{L,R}^\phi$  ( $\phi = h, H, A, H^-$ ) are the  $\phi$  contributions at the one loop level. The Yukawa couplings  $y_{\phi\tau\tau}^e$  ( $\phi = h, H, A$ ) are given in Eq. (7). Here we have neglected

<sup>2</sup> Yukawa couplings  $y_{\phi\mu\mu}^e$  also contribute to  $\tau \rightarrow \mu\gamma$ . However, the SM part of  $y_{\phi\mu\mu}^e$  is smaller than the one of  $y_{\phi\tau\tau}^e$ , and  $\rho_e^{\mu\mu}$  is strongly constrained by  $\tau \rightarrow 3\mu$  process as discussed later. Therefore we have neglected the contributions from  $y_{\phi\mu\mu}^e$ .

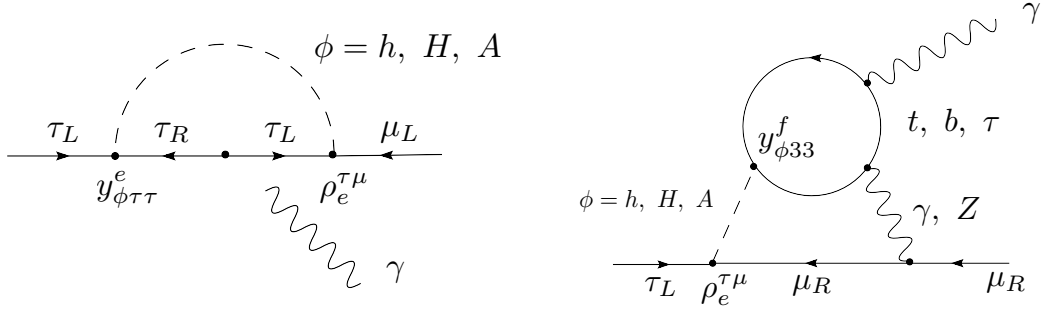


FIG. 5: Some of Feynman diagrams which contribute to  $\tau \rightarrow \mu\gamma$  processes at one-loop level (left figure) which are induced by  $y_{\phi\tau\tau}^e$  ( $\phi = h, H, A$ ) Yukawa couplings and at two-loop level (right figure) which are Barr-Zee type contributions and induced by the third generation fermions via  $y_{\phi 33}^f$  ( $f = u, d, e$ ) Yukawa couplings. Diagrams where the fermion chiralities are flipped also contribute. We also have a Barr-Zee type two-loop contribution induced by W-loop, which is not shown here.

the  $O(m_\mu/m_\tau)$  contributions.

We also find that the Barr-Zee type contributions ( $A_{L,R}^{\text{BZ}}$ ) at the two loop level are important and dominant in the most of cases. The third generation fermion contributions via  $y_{\phi 33}^f$  ( $f = u, d, e$ ) Yukawa couplings<sup>3</sup> (shown in the right figure

<sup>3</sup> In our notation,  $y_{\phi 33}^u = y_{\phi tt}^u$ ,  $y_{\phi 33}^d = y_{\phi bb}^d$ ,  $y_{\phi 33}^e = y_{\phi \tau\tau}^e$ .

in Fig. 5) and the W-boson contribution (not shown in Fig. 5) are given by <sup>4</sup>

$$\begin{aligned}
A_L^{\text{BZ}} = & - \sum_{\phi=h,H,A;f=u,d,e} \frac{N_C Q_f \alpha}{8\pi^3} \frac{y_{\phi\tau\mu}^{e*}}{m_\tau m_{f_3}} \left[ Q_f \left\{ \text{Re}(y_{\phi 33}^f) F_H(x_{f\phi}) - i \text{Im}(y_{\phi 33}^f) F_A(x_{f\phi}) \right\} \right. \\
& + \frac{(1-4s_W^2)(2T_{3f}-4Q_f s_W^2)}{16s_W^2 c_W^2} \left\{ \text{Re}(y_{\phi 33}^f) \tilde{F}_H(x_{f\phi}, x_{fZ}) - i \text{Im}(y_{\phi 33}^f) \tilde{F}_A(x_{f\phi}, x_{fZ}) \right\} \Big] \\
& + \sum_{\phi=h,H} \frac{\alpha}{16\pi^3} \frac{g_{\phi WW} y_{\phi\tau\mu}^{e*}}{m_\tau v} \left[ 3F_H(x_{W\phi}) + \frac{23}{4} F_A(x_{W\phi}) \right. \\
& \quad \left. + \frac{3}{4} G(x_{W\phi}) + \frac{m_\phi^2}{2m_W^2} \{F_H(x_{W\phi}) - F_A(x_{W\phi})\} \right. \\
& \quad + \frac{1-4s_W^2}{8s_W^2} \left\{ \left( 5 - t_W^2 + \frac{1-t_W^2}{2x_{W\phi}} \right) \tilde{F}_H(x_{W\phi}, x_{WZ}) \right. \\
& \quad \left. + \left( 7 - 3t_W^2 - \frac{1-t_W^2}{2x_{W\phi}} \right) \tilde{F}_A(x_{W\phi}, x_{WZ}) + \frac{3}{2} \{F_A(x_{W\phi}) + G(x_{W\phi})\} \right\} \Big], \quad (20)
\end{aligned}$$

$$A_R^{\text{BZ}} = A_L^{\text{BZ}}(y_{\phi\tau\mu}^{e*} \rightarrow y_{\phi\mu\tau}^e, i \rightarrow -i), \quad (21)$$

where  $x_{f\phi} = m_{f_3}^2/m_\phi^2$ ,  $x_{fZ} = m_{f_3}^2/m_Z^2$  ( $f_3 = t, b, \tau$  for  $f = u, d, e$ ),  $x_{W\phi} = m_W^2/m_\phi^2$  and  $x_{WZ} = m_W^2/m_Z^2$ , and  $s_W^2 = \sin^2 \theta_W$ ,  $c_W^2 = \cos^2 \theta_W$  and  $t_W^2 = \tan^2 \theta_W$ .  $T_{3f}$  denotes the isospin of the fermion. Here the couplings  $g_{\phi WW} = s_{\beta\alpha}$  ( $c_{\beta\alpha}$ ) for  $\phi = h$  ( $\phi = H$ ). Functions  $F_H, A, G$  and  $\tilde{F}_H, A$  are defined by

$$F_H(z) = \frac{z}{2} \int_0^1 dx \frac{1-2x(1-x)}{x(1-x)-z} \log \frac{x(1-x)}{z}, \quad (22)$$

$$F_A(z) = \frac{z}{2} \int_0^1 dx \frac{1}{x(1-x)-z} \log \frac{x(1-x)}{z}, \quad (23)$$

$$G(z) = -\frac{z}{2} \int_0^1 dx \frac{1}{x(1-x)-z} \left[ 1 - \frac{z}{x(1-x)-z} \log \frac{x(1-x)}{z} \right], \quad (24)$$

$$\tilde{F}_H(x, y) = \frac{x F_H(y) - y F_H(x)}{x - y}, \quad (25)$$

$$\tilde{F}_A(x, y) = \frac{x F_A(y) - y F_A(x)}{x - y}. \quad (26)$$

Note if the Yukawa couplings  $\rho_f^{ij}$  are real,  $\text{Im}(y_{\phi 33}^f) = 0$  for  $f = h$  and  $H$ , and  $\text{Re}(y_{\phi 33}^f) = 0$  for  $f = A$  are satisfied, as shown in Eq. (7). For simplicity, we assume that all  $\rho_f^{ij}$  are real in the calculation of  $\tau \rightarrow \mu\gamma$ . The contribution in the first line (the second line) of Eq. (20) comes from the effective  $\phi\gamma\gamma$  vertex ( $\phi Z\gamma$  vertex)

<sup>4</sup> The Barr-Zee contributions to  $\mu \rightarrow e\gamma$  have been studied in Ref. [44]. The application to  $\tau \rightarrow \mu\gamma$  is apparent and we adopt their results for  $\tau \rightarrow \mu\gamma$ .

induced by the third generation fermion loop, and the one in the third and the forth lines (the fifth and sixth lines) originates from the effective  $\phi\gamma\gamma$  vertex ( $\phi Z\gamma$  vertex) generated by the W-boson loop. In the analysis of  $\tau \rightarrow \mu\gamma$  in Ref. [14], we have not included the Barr-Zee type contributions induced by the effective  $\phi\gamma Z$  vertex since they are sub-dominant contributions. Here we include them and find they change the results by about 10%.

The total amplitude  $A_{L, R}$  is a sum of all contributions,

$$A_{L, R} = \sum_{\phi=h,H,A,H^-} A_{L, R}^{\phi} + A_{L, R}^{\text{BZ}}. \quad (27)$$

In Fig. 6, numerical results for  $\text{BR}(\tau \rightarrow \mu\gamma)$  as a function of  $c_{\beta\alpha}$  and  $\text{BR}(h \rightarrow \mu\tau)$  are shown. Here the same parameter set as the one in Fig. 2 is taken. We have assumed that the extra Yukawa couplings  $\rho_f^{ij}$  other than  $\rho_e^{\mu\tau(\tau\mu)}$  are negligible. Lines for  $\text{BR}(\tau \rightarrow \mu\gamma)/10^{-9} = 0.4, 0.7, 1.0$  and  $1.3$  (upper figure) and for  $\text{BR}(\tau \rightarrow \mu\gamma)/10^{-9} = 0.5, 1.0$  and  $1.5$  (lower figure) are shown for  $m_A = 250$  GeV and  $350$  GeV, respectively. As one can see, if  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  as suggested by the CMS experiment, the branching ratio for  $\tau \rightarrow \mu\gamma$  can be larger than  $10^{-9}$ , which might be within the reach of the future B-factory experiment, the Belle II. Note that the branching ratio  $\text{BR}(\tau \rightarrow \mu\gamma)$  almost does not depend on the Higgs mixing parameter  $c_{\beta\alpha}$  when  $\text{BR}(h \rightarrow \mu\tau)$  is fixed and  $c_{\beta\alpha}$  is small. We also note that the cancellation between the one-loop and two-loop Barr-Zee type contributions happens, and hence the branching ratio  $\text{BR}(\tau \rightarrow \mu\gamma)$  is not simply suppressed by the heavy Higgs boson masses.

If the extra Yukawa couplings other than  $\rho_e^{\mu\tau(\tau\mu)}$  are not negligible, the branching ratio  $\text{BR}(\tau \rightarrow \mu\gamma)$  could be further enhanced. For example, the extra Yukawa coupling  $\rho_e^{\tau\tau}$  can contribute at the one-loop level, and on the other hand,  $\rho_u^{tt}$  can affect the branching ratio via the Barr-Zee type two-loop contribution. In Fig. 7, numerical results for  $\text{BR}(\tau \rightarrow \mu\gamma)$  are shown as a function of  $\rho_e^{\tau\tau}$  and  $\rho_u^{tt}$ . Lines for  $\text{BR}(\tau \rightarrow \mu\gamma)/10^{-8} = 0.1$  and  $4.4$  (current experimental limit) are shown. Here we have assumed  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$ ,  $c_{\beta\alpha} = -0.007$  and  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ . This parameter set can enhance the muon g-2 as  $\delta a_\mu = 2.2 \times 10^{-9}$  which is within the  $1\sigma$ . At present, the extra Yukawa couplings  $\rho_e^{\tau\tau}$  and  $\rho_u^{tt}$  can be still larger than, for example,  $O(0.1)$  with some correlation, however,

the future experimental constraint would be significant for this scenario. Therefore, the  $\tau \rightarrow \mu\gamma$  process would be important to probe the scenario.

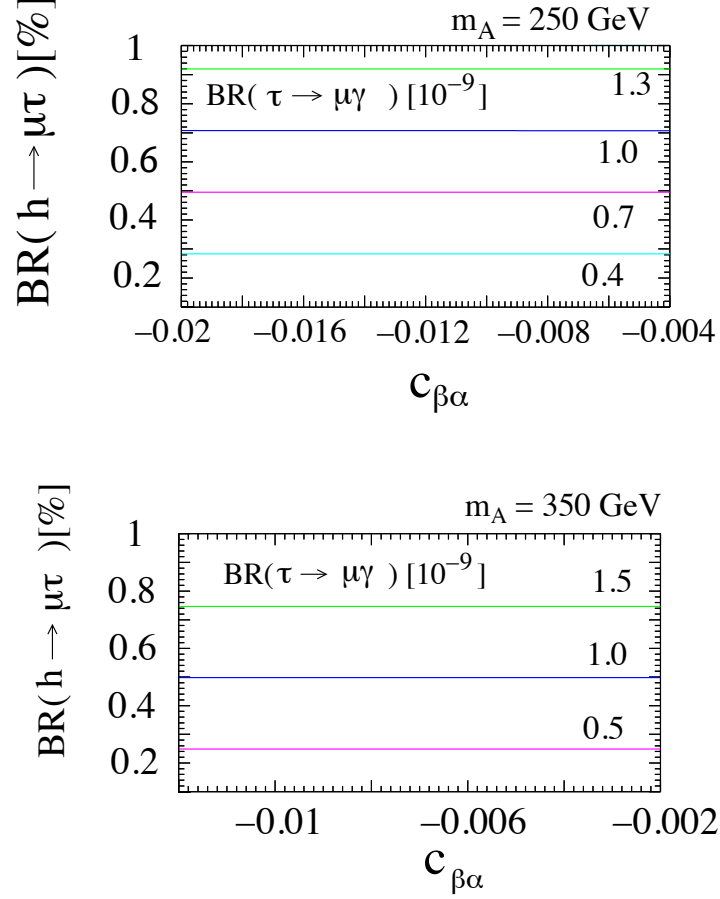


FIG. 6: Numerical result for  $\text{BR}(\tau \rightarrow \mu\gamma)$  as a function of  $c_{\beta\alpha}$  and  $\text{BR}(h \rightarrow \mu\tau)$  in the same parameter set of Fig. 2. Lines for  $\text{BR}(\tau \rightarrow \mu\gamma)/10^{-9} = 0.4, 0.7, 1.0$  and  $1.3$  ( $0.5, 1.0$  and  $1.5$ ) are shown for  $m_A = 250 \text{ GeV}$  ( $m_A = 350 \text{ GeV}$ ). Here we have assumed that the extra Yukawa couplings  $\rho_f$  other than  $\rho_e^{\mu\tau} (\tau\mu)$  are negligible.

### B. $\mu \rightarrow e\gamma$ , $\tau \rightarrow e\gamma$ , and electron g-2

The  $\mu - \tau$  flavor violating Yukawa couplings themselves do not generate  $\mu \rightarrow e\gamma$ . However, together with  $e - \mu$  or  $e - \tau$  flavor-violation,  $\mu \rightarrow e\gamma$  is induced. Since there is a strong constraint from this process,  $e - \mu$  and  $e - \tau$  flavor violating couplings



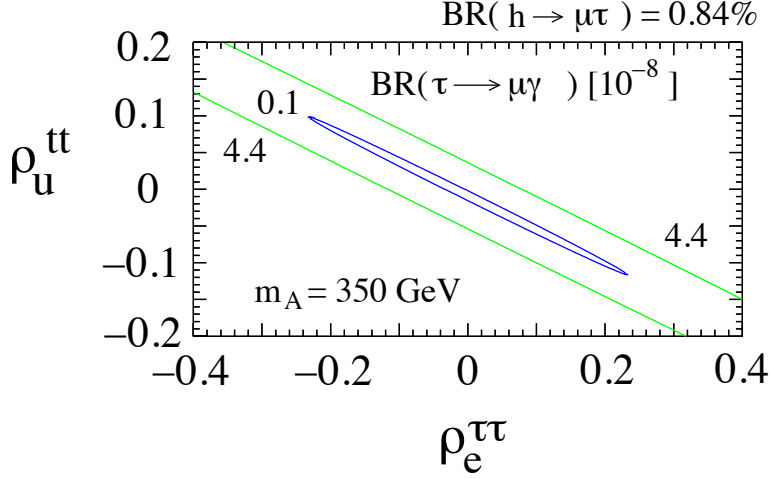


FIG. 7: Numerical result for  $\text{BR}(\tau \rightarrow \mu\gamma)$  as a function of  $\rho_e^{\tau\tau}$  and  $\rho_u^{tt}$ . Lines for  $\text{BR}(\tau \rightarrow \mu\gamma)/10^{-8} = 0.1$  and  $4.4$  (current experimental limit) are shown. Here we have assumed  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$ ,  $c_{\beta\alpha} = -0.007$  and  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ . We note that for this parameter set,  $\delta a_\mu = 2.2 \times 10^{-9}$  which explains the muon g-2 anomaly within the  $1\sigma$ .

are strongly constrained.

Similar to  $\tau \rightarrow \mu\gamma$ , we parametrize the decay amplitude ( $T_{\mu \rightarrow e\gamma}$ ) as

$$T_{\mu \rightarrow e\gamma} = e\epsilon^{\alpha*} \bar{u}_e m_\mu i\sigma_{\alpha\beta} q^\beta (A_L P_L + A_R P_R) u_\mu, \quad (28)$$

and the branching ratio is given by

$$\text{BR}(\mu \rightarrow e\gamma) = \frac{48\pi^3 \alpha (|A_L|^2 + |A_R|^2)}{G_F^2}. \quad (29)$$

The neutral Higgs contributions  $A_{L,R}^\phi$  ( $\phi = h, H, A$ ) to  $A_{L,R}$  at the one-loop are given by

$$A_L^\phi = \frac{1}{16\pi^2} \sum_{i=\mu,\tau} \frac{y_{\phi ie}^{e*}}{m_\phi^2} \left[ \frac{m_i}{m_\mu} y_{\phi i\mu}^{e*} \left( \log \frac{m_\phi^2}{m_i^2} - \frac{3}{2} \right) + \frac{y_{\phi i\mu}^e}{6} \right], \quad (30)$$

$$A_R^\phi = \frac{1}{16\pi^2} \sum_{i=\mu,\tau} \frac{y_{\phi ei}^e}{m_\phi^2} \left[ \frac{m_i}{m_\mu} y_{\phi i\mu}^e \left( \log \frac{m_\phi^2}{m_i^2} - \frac{3}{2} \right) + \frac{y_{\phi i\mu}^{e*}}{6} \right], \quad (31)$$

where the Yukawa couplings  $y_{\phi ij}^e$  are defined in Eq. (7). Here we neglect an electron

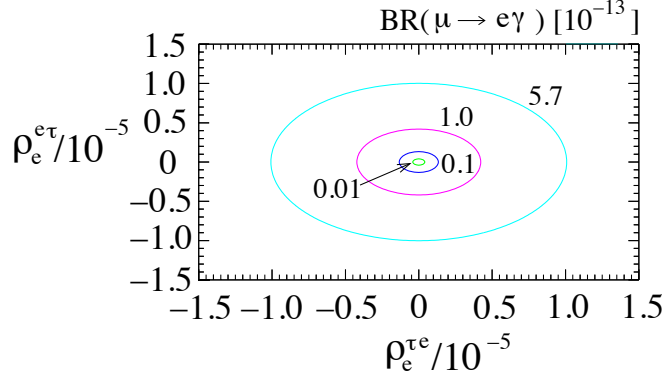


FIG. 8: Numerical result for  $\text{BR}(\mu \rightarrow e\gamma)$  as a function of  $\rho_e^{\tau e}$  and  $\rho_e^{e\tau}$ . Lines for  $\text{BR}(\mu \rightarrow e\gamma)/10^{-13} = 5.7$  (current limit), 1.0, 0.1 and 0.01 are shown. Here we have assumed that  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$ ,  $c_{\beta\alpha} = -0.007$  and  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ , and extra Yukawa couplings  $\rho_f$  other than  $\rho_e^{\mu\tau(\tau\mu)}$  and  $\rho_e^{\tau e(e\tau)}$  are negligible. We note that for this parameter set,  $\delta a_\mu = 2.2 \times 10^{-9}$ .

mass and we assume that the Yukawa coupling  $y_{\phi ee}^e$  is negligible.<sup>5</sup> The charged Higgs contribution to  $A_{L,R}$  is

$$A_L^{H^-} = -\frac{(\bar{\rho}_e^\dagger \rho_e)_{e\mu}}{192\pi^2 m_{H^-}^2}, \quad A_R^{H^-} = 0. \quad (32)$$

For nonzero  $y_{\phi \mu e}^e$ , the Barr-Zee type contributions ( $A_{L,R}^{\text{BZ}}$ ) at two-loop level are significant. The expression of  $A_{L,R}^{\text{BZ}}$  is the same as one for  $\tau \rightarrow \mu\gamma$  case shown in Eq. (20) except that the flavor violating Yukawa couplings  $y_{\phi \tau \mu}^{e(*)}$  should be replaced by  $y_{\phi \mu e}^{e(*)}$ , and the  $\tau$  mass ( $m_\tau$ ) should be replaced by the  $\mu$  mass ( $m_\mu$ ). The total  $A_{L,R}$  is a sum of all contributions;

$$A_{L,R} = \sum_{\phi=h,H,A,H^-} A_{L,R}^\phi + A_{L,R}^{\text{BZ}}. \quad (33)$$

Similar to the muon g-2, the contributions from the  $\mu - \tau$  flavor violating Yukawa interactions together with the  $e - \tau$  flavor violation have  $O(m_\tau/m_\mu)$  enhancement, and induce significant contributions to  $\mu \rightarrow e\gamma$ . In Fig. 8, we show numerical results

<sup>5</sup> The Yukawa coupling  $\rho_e^{ee}$  is strongly constrained by  $\tau \rightarrow \mu e^+ e^-$  process, as studied later. Therefore, our assumption will be justified.

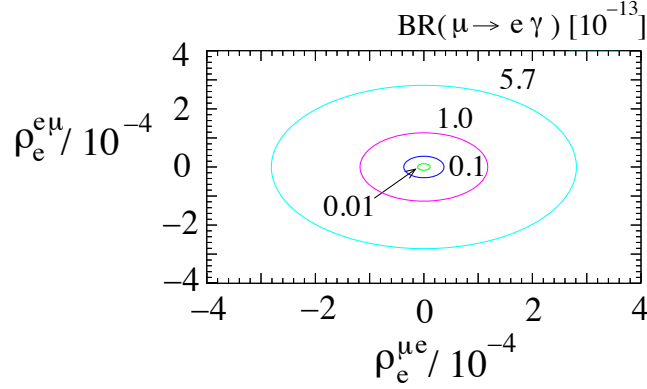


FIG. 9: Numerical result for  $\text{BR}(\mu \rightarrow e\gamma)$  as a function of  $\rho_e^{\mu e}$  and  $\rho_e^{e\mu}$ . Lines for  $\text{BR}(\mu \rightarrow e\gamma)/10^{-13} = 5.7$  (current limit), 1.0, 0.1 and 0.01 are shown. Here we have assumed that  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$ ,  $c_{\beta\alpha} = -0.007$ , and extra Yukawa couplings  $\rho_f$  other than  $\rho_e^{\mu\tau(\tau\mu)}$  and  $\rho_e^{\mu e(e\mu)}$  are negligible.

for  $\text{BR}(\mu \rightarrow e\gamma)$  as a function of  $\rho_e^{\tau e}$  and  $\rho_e^{e\tau}$ . Here we have taken  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$ ,  $c_{\beta\alpha} = -0.07$  and  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ . We have assumed that extra Yukawa couplings  $\rho_f^{ij}$  other than  $\rho_e^{\mu\tau(\tau\mu)}$  and  $\rho_e^{\tau e(e\tau)}$  are negligible. This parameter set corresponds to  $\delta a_\mu = 2.2 \times 10^{-9}$ . One can see that the current limit on  $\text{BR}(\mu \rightarrow e\gamma)$  strongly constrains the  $e - \tau$  flavor violating couplings  $\rho_e^{\tau e(e\tau)}$  if the CMS excess of  $\text{BR}(h \rightarrow \mu\tau)$  is true. If we change the value of  $\text{BR}(h \rightarrow \mu\tau)$  in Fig. 8, the experimental bound of  $\rho_e^{\tau e(e\tau)}$  is relaxed by the factor,  $\sqrt{\frac{0.84\%}{\text{BR}(h \rightarrow \mu\tau)}}$  when  $\rho_e^{\tau e} = \rho_e^{e\tau}$  is assumed.

If Yukawa couplings  $\rho_e^{\tau e(e\tau)}$  are negligible but  $\rho_e^{\mu e(e\mu)}$  are not, the Barr-Zee type two loop contributions are dominant.<sup>6</sup> We show numerical results for  $\text{BR}(\mu \rightarrow e\gamma)$  as a function of  $\rho_e^{\mu e}$  and  $\rho_e^{e\mu}$ . Here we have assumed that  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$ ,  $c_{\beta\alpha} = -0.007$  and extra Yukawa couplings  $\rho_f^{ij}$  other than  $\rho_e^{\mu\tau(\tau\mu)}$  and  $\rho_e^{\mu e(e\mu)}$  are negligible. As one can see from the figure,  $\rho_e^{\mu e(e\mu)}$  couplings are also severely constrained by the  $\mu \rightarrow e\gamma$  bound. Note that the prediction of  $\mu \rightarrow e\gamma$  for

<sup>6</sup> If  $\rho_e^{\mu\mu}$  is also nonzero, there are also one-loop contributions as shown in Eq. (31). However, the coupling  $\rho_e^{\mu\mu}$  is strongly constrained by the  $\tau \rightarrow 3\mu$  bound, as discussed later. Therefore, the effect from  $\rho_e^{\mu\mu}$  is negligible and we neglect it in our numerical analysis.

this case does not depend on the value of  $\text{BR}(h \rightarrow \mu\tau)$ . The future improvement of  $\text{BR}(\mu \rightarrow e\gamma)$  at the level of  $10^{-14}$  as proposed by the MEG II experiment [45] would significantly probe the flavor structure of this scenario.

The effective operator for  $\mu \rightarrow e\gamma$  also generate  $\mu - e$  conversion process in nuclei. Besides, the extra Yukawa couplings,  $\rho_e^{\mu e}$  and  $\rho_e^{e\mu}$ , may enhance the  $\mu - e$  conversion through the tree-level Higgs exchanging. The contribution depends on the extra Yukawa couplings in the quark sector as well, and then our model may be also tested by the experiments [46–49], although our prediction is vague because of the ambiguity of the Yukawa couplings.<sup>7</sup>

We comment on the consequence of the strong constraints on the  $e - \tau$  and  $e - \mu$  flavor-violations. Unlike the  $\mu - \tau$  flavor violation, the  $e - \tau$  flavor violating Yukawa couplings in this scenario is strongly constrained as we have seen above. Therefore, the prediction of  $\text{BR}(\tau \rightarrow e\gamma)$  is expected to be small. Similarly, because of the smallness of the  $e - \tau$  and  $e - \mu$  flavor violation, we also expect that the new physics contributions to the anomalous magnetic moment of electron (electron g-2) should be small.

### C. Muon electric dipole moment (muon EDM)

When we discussed the muon g-2, we have assumed that the  $\mu - \tau$  flavor violating Yukawa couplings are real. If the  $\mu - \tau$  Yukawa couplings are complex, the couplings  $\rho_e^{\mu\tau} \rho_e^{\tau\mu}$  in Eq. (15) should be replaced by  $\text{Re}(\rho_e^{\mu\tau} \rho_e^{\tau\mu})$ . In addition, the imaginary parts of the Yukawa couplings generate an electric dipole moment (EDM) of muon. Since the muon g-2 and the muon EDM are induced by the same Feynman diagram shown in Fig. 1, these quantities are correlated via the unknown CP-violating phase. The effective operators for the muon g-2 ( $\delta a_\mu$ ) and the muon EDM ( $\delta d_\mu$ ) are expressed by

$$\mathcal{L} = \bar{\mu}\sigma^{\mu\nu} \left( \frac{e}{4m_\mu} \delta a_\mu - \frac{i}{2} \delta d_\mu \gamma_5 \right) \mu F_{\mu\nu}. \quad (34)$$

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<sup>7</sup> The study on the tree-level flavor changing couplings of quarks is beyond our scope.

If we parametrize the complex Yukawa couplings as follows:

$$\rho_e^{\mu\tau} \rho_e^{\tau\mu} = |\rho_e^{\mu\tau} \rho_e^{\tau\mu}| e^{i\phi}, \quad (35)$$

the relation between the muon g-2 ( $\delta a_\mu$ ) and the muon EDM ( $\delta d_\mu$ ) induced by the  $\mu - \tau$  flavor-violating Yukawa couplings is given by

$$\frac{\delta d_\mu}{\delta a_\mu} = -\frac{e \tan \phi}{2m_\mu}. \quad (36)$$

Therefore, the predicted muon EDM is

$$\delta d_\mu = -3 \times 10^{-22} \text{ e} \cdot \text{cm} \times \left( \frac{\tan \phi}{1.0} \right) \left( \frac{\delta a_\mu}{3 \times 10^{-9}} \right). \quad (37)$$

The current limit [50] is

$$|d_\mu| < 1.9 \times 10^{-19} \text{ e} \cdot \text{cm} \text{ (95\% C.L.)}, \quad (38)$$

and hence it is not sensitive to this scenario at present. However, the future improvement at the level of  $10^{-24} \text{ e} \cdot \text{cm}$  [51] would be significant to probe the scenario.

#### D. $\tau \rightarrow \mu\nu\bar{\nu}$

The Yukawa couplings  $\rho_e^{\mu\tau(\tau\mu)}$  induce a correction to  $\tau \rightarrow \mu\nu\bar{\nu}$  via a charged Higgs mediation, where the flavor of final neutrino and anti-neutrino states is summed up since it is not detected.<sup>8</sup>

The correction  $\delta$  is given as follows;

$$\begin{aligned} \Gamma(\tau \rightarrow \mu\nu\bar{\nu}) &= \frac{m_\tau^5 G_F^2}{192\pi^3} (1 + \delta), \\ \delta &= \frac{|\rho_e^{\mu\tau}|^2 |\rho_e^{\tau\mu}|^2}{32 G_F^2 m_{H^+}^4}. \end{aligned} \quad (39)$$

In Fig. 10, numerical results for the correction  $\delta$  given above are shown as a function of  $c_{\beta\alpha}$  and  $\text{BR}(h \rightarrow \mu\tau)$  in the same parameter set of Fig. 2. One can see that as

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<sup>8</sup> In general, the unknown Yukawa couplings  $\rho_e^{i\tau}$  and  $\rho_e^{i\mu}$  ( $i = e, \mu, \tau$ ) generate the extra corrections to  $\delta$ . However, the Yukawa couplings  $\rho_e^{e\tau}$  ( $e\mu$ ) and  $\rho_e^{\mu\mu}$  are strongly constrained by  $\mu \rightarrow e\gamma$  and  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$ , respectively. Therefore, the contributions from these couplings are negligible. The unknown Yukawa coupling  $\rho_e^{\tau\tau}$  can be sizable, and hence it can increase the prediction of the  $\delta$ . Thus our result of  $\delta$  induced from  $\rho_e^{\mu\tau(\tau\mu)}$  is viewed as a conservative estimate.

the correction to the muon g-2 ( $\delta a_\mu$ ) gets larger, the size of  $\delta$  also becomes larger, and they are correlated each other, independent of  $\text{BR}(h \rightarrow \mu\tau)$ . The interesting regions which explain the muon g-2 anomaly within  $1\sigma$  predict  $\delta \leq 10^{-4}$ - $10^{-3}$ . The current precision of the measurement of the decay rate  $\Gamma(\tau \rightarrow \mu\nu\bar{\nu})$  is at the level of  $10^{-3}$  [40]. Therefore, the further improvement of the precision would be important for this scenario. In addition, from the  $\tau$  decay, the BaBar collaboration has reported a measurement of the charged current lepton universality [52], given by

$$\left(\frac{g_\mu}{g_e}\right)^2 = \frac{\text{BR}(\tau^- \rightarrow \mu^- \nu\bar{\nu})}{\text{BR}(\tau^- \rightarrow e^- \nu\bar{\nu})} \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}, \quad (40)$$

where  $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$ , which is a phase space factor. The universality of the gauge interaction in the SM predicts  $g_e = g_\mu$  and the current experimental results are

$$\begin{aligned} \left(\frac{g_\mu}{g_e}\right) &= 1.0036 \pm 0.0020 \text{ (BaBar)}, \\ &= 1.0018 \pm 0.0014 \text{ (world average)}. \end{aligned} \quad (41)$$

In our scenario, we expect the correction to  $\tau \rightarrow e\nu\bar{\nu}$  would be small because of the strong constraint on  $e - \tau$  flavor violation from  $\mu \rightarrow e\gamma$  process. Therefore, the charged Higgs contribution to  $\tau \rightarrow \mu\nu\bar{\nu}$  with  $\mu - \tau$  flavor violating Yukawa couplings induces the significant correction to the violation of the lepton universality above,

$$\left(\frac{g_\mu}{g_e}\right)^2 = 1 + \delta. \quad (42)$$

The result from the Belle collaboration and the further improvement of the precision of the lepton universality would have an important impact on our scenario.

#### **E. $\tau^- \rightarrow \mu^- l^+ l^-$ , $\tau^- \rightarrow e^- l^+ l^-$ ( $l = e, \mu$ ), $\mu^+ \rightarrow e^+ e^- e^+$ and others**

The nonzero Yukawa couplings  $\rho_e^{\mu\tau(\tau\mu)}$  also generate processes  $\tau^- \rightarrow \mu^- \mu^+ \mu^-$  and  $\tau^- \rightarrow \mu^- e^+ e^-$  ( $\tau \rightarrow 3\mu$  and  $\tau \rightarrow \mu ee$  for short, respectively) at the tree level. They are induced without unknown  $\rho_e^{\mu\mu}$  and  $\rho_e^{ee}$  Yukawa couplings. The branching ratios, however, are too small to be observed. Therefore, nonzero  $\rho_e^{\mu\mu}$  and  $\rho_e^{ee}$  are important

for these processes<sup>9</sup>. The branching ratios for  $\tau \rightarrow 3\mu$  and  $\tau \rightarrow \mu ee$  are given by [53]

$$\frac{\text{BR}(\tau \rightarrow 3\mu)}{\text{BR}(\tau \rightarrow \mu\nu\bar{\nu})} = \sum_{\phi, \phi'=h, H, A} \frac{I(\phi, \phi')}{64G_F^2},$$

$$I(\phi, \phi') = 2 \left( \frac{y_{\phi\mu\tau}^e y_{\phi\mu\mu}^{e*}}{m_\phi^2} \right) \left( \frac{y_{\phi'\mu\tau}^{e*} y_{\phi'\mu\mu}^e}{m_{\phi'}^2} \right) + 2 \left( \frac{y_{\phi\tau\mu}^e y_{\phi\mu\mu}^{e*}}{m_\phi^2} \right) \left( \frac{y_{\phi'\tau\mu}^{e*} y_{\phi'\mu\mu}^e}{m_{\phi'}^2} \right) \\ + \left( \frac{y_{\phi\mu\tau}^e y_{\phi\mu\mu}^e}{m_\phi^2} \right) \left( \frac{y_{\phi'\mu\tau}^{e*} y_{\phi'\mu\mu}^{e*}}{m_{\phi'}^2} \right) + \left( \frac{y_{\phi\tau\mu}^e y_{\phi\mu\mu}^e}{m_\phi^2} \right) \left( \frac{y_{\phi'\tau\mu}^{e*} y_{\phi'\mu\mu}^{e*}}{m_{\phi'}^2} \right), \quad (43)$$

$$\frac{\text{BR}(\tau \rightarrow \mu ee)}{\text{BR}(\tau \rightarrow \mu\nu\bar{\nu})} = \sum_{\phi, \phi'=h, H, A} \frac{J(\phi, \phi')}{32G_F^2},$$

$$J(\phi, \phi') = \left( \frac{y_{\phi\mu\tau}^e y_{\phi ee}^{e*}}{m_\phi^2} \right) \left( \frac{y_{\phi'\mu\tau}^{e*} y_{\phi' ee}^e}{m_{\phi'}^2} \right) + \left( \frac{y_{\phi\tau\mu}^e y_{\phi ee}^{e*}}{m_\phi^2} \right) \left( \frac{y_{\phi'\tau\mu}^{e*} y_{\phi' ee}^e}{m_{\phi'}^2} \right) \\ + \left( \frac{y_{\phi\mu\tau}^e y_{\phi ee}^e}{m_\phi^2} \right) \left( \frac{y_{\phi'\mu\tau}^{e*} y_{\phi' ee}^{e*}}{m_{\phi'}^2} \right) + \left( \frac{y_{\phi\tau\mu}^e y_{\phi ee}^e}{m_\phi^2} \right) \left( \frac{y_{\phi'\tau\mu}^{e*} y_{\phi' ee}^{e*}}{m_{\phi'}^2} \right). \quad (44)$$

Fig. 11 shows  $\text{BR}(\tau \rightarrow 3\mu)$  and  $\text{BR}(\tau \rightarrow \mu ee)$  as a function of  $\rho_e^{ll}$  ( $l = \mu$  for  $\tau \rightarrow 3\mu$  and  $l = e$  for  $\tau \rightarrow \mu ee$ ). It is assumed that  $c_{\beta\alpha} = -0.007$ ,  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$  and  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$  in Fig. 11. One can see that the current experimental bounds,

$$\text{BR}(\tau \rightarrow 3\mu) < 2.1 \times 10^{-8}, \quad \text{BR}(\tau \rightarrow \mu ee) < 1.8 \times 10^{-8} \quad (45)$$

set the strong constraints on the  $\rho_e^{ll}$  Yukawa couplings. For example, the parameter set shown in Fig. 11, requires  $\rho_e^{ll} < 0.006$  ( $l = \mu, e$ ). We note that the constraint on the  $\rho_e^{\mu\mu}$  is still larger than the value of the muon Yukawa coupling in the SM ( $y_\mu = \frac{\sqrt{2}m_\mu}{v} \sim 6 \times 10^{-4}$ ).

Contrary to  $\tau^- \rightarrow \mu^- l^+ l^-$ , the  $\tau^- \rightarrow e^- l^+ l^-$  ( $l = e, \mu$ ) process is suppressed in this scenario because the  $\tau - e$  flavor violation is strongly constrained by  $\mu \rightarrow e\gamma$  process. Furthermore, since the constraints on  $\rho_e^{\mu\mu} (\mu e)$  are stronger than those on  $\rho_e^{\mu\mu(ee)}$ ,  $\tau^- \rightarrow \mu^- e^+ \mu^-$  is expected to be smaller than  $\tau^- \rightarrow \mu^- l^+ l^-$ . (Needless to say,  $\tau^- \rightarrow e^- \mu^+ e^-$  is much suppressed.) Therefore, we expect that the  $\tau^- \rightarrow e^- l^+ l^-$  and  $\tau^- \rightarrow \mu^- e^+ \mu^-$  processes will be small.

<sup>9</sup> Nonzero  $\rho_e^{\tau e(e\tau)}$  and  $\rho_e^{\mu e(e\mu)}$  couplings also induce the  $\tau \rightarrow \mu ee$  process. However, these Yukawa couplings are strongly constrained by  $\mu \rightarrow e\gamma$  process as discussed in previous sections. Therefore, we neglect these effects.

We also study  $\mu^+ \rightarrow e^+e^-e^+$  ( $\mu \rightarrow 3e$  in short) which depends on the  $\mu - e$  flavor violating Yukawa couplings  $\rho_e^{e\mu(\mu e)}$  and the flavor diagonal element  $\rho_e^{ee}$ . As we have seen, the  $\mu - e$  flavor violating Yukawa couplings  $\rho_e^{e\mu(\mu e)}$  are constrained by the  $\mu \rightarrow e\gamma$  process and the  $\rho_e^{ee}$  coupling is restricted by the  $\tau \rightarrow \mu ee$  process. From Fig. 9 and Fig. 11, the current limits on  $\rho_e^{\mu e(e\mu)}$  and  $\rho_e^{ee}$  are  $\rho_e^{\mu e} < 2 \times 10^{-4}$  for  $\rho_e^{\mu e} = \rho_e^{e\mu}$  and  $\rho_e^{ee} < 6 \times 10^{-3}$ , respectively, assuming  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$  and  $c_{\beta\alpha} = -0.007$ . Under these constraints, it will be interesting to see how large branching ratio of  $\mu \rightarrow 3e$  is expected. In Fig. 12, we show the  $\text{BR}(\mu \rightarrow 3e)$  as a function of  $\rho_e^{ee}$  and  $\rho_e^{\mu e}$ . In the parameter region where the constraints from  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu ee$  are satisfied, the branching ratio can be as large as about  $10^{-13}$ . This is consistent with the current limit [40]

$$\text{BR}(\mu \rightarrow 3e) < 1.0 \times 10^{-12}. \quad (46)$$

The improvement of the branching ratio at the level of  $10^{-16}$  [54] which has been proposed by the Mu3e experiment would have a significant impact on this scenario together with the improvement of  $\mu \rightarrow e\gamma$  [45] and  $\mu - e$  conversion in nuclei [46–49].

#### F. $\tau \rightarrow \mu\eta$

The  $\tau \rightarrow \mu\eta$  is also generated by the extra  $\rho_d^{ss}$  Yukawa coupling via the mediation of the CP-odd Higgs boson at the tree level. The expression for the branching ratio of  $\tau \rightarrow \mu\eta$  is given by [55, 56]

$$\text{BR}(\tau \rightarrow \mu\eta) = \frac{3|\rho_d^{ss}|^2(\bar{\rho}^{\mu\tau})^2}{32\pi} \frac{m_\tau F_\eta^2}{m_A^4 \Gamma_\tau} \left( \frac{m_\eta^2}{m_u + m_d + 4m_s} \right)^2 \left( 1 - \frac{m_\eta^2}{m_\tau^2} \right)^2, \quad (47)$$

where  $m_\eta$  and  $F_\eta$  are the mass and the decay constant of  $\eta$ . For  $F_\eta = 150$  MeV and  $m_\eta = 548$  MeV, we obtain a constraint on  $\rho_d^{ss}$ ;

$$|\rho_d^{ss}| < 0.007 \left( \frac{0.3}{\bar{\rho}^{\mu\tau}} \right) \left( \frac{m_A}{350 \text{ GeV}} \right)^2. \quad (48)$$

We have a strong constraint although it is still larger than the SM value of the strange quark Yukawa coupling ( $y_s = \frac{\sqrt{2}m_s}{v} \sim 5 \times 10^{-4}$ ).

The other hadronic  $\tau$ -lepton decays have been studied in Ref. [57]. They potentially provide constraints on the other extra Yukawa couplings  $\rho_f$  in quark sector. For details, see Ref. [57].



## V. IMPLICATION TO HIGGS PHYSICS

We have seen that the CMS excess in  $h \rightarrow \mu\tau$  is consistent with the anomaly of muon  $g-2$  as well as the other experimental constraints. It will be interesting to note whether other lepton flavor violating Higgs boson decays would be possible. As we have already seen, the  $e - \mu$  and  $e - \tau$  flavor violating Yukawa couplings are strongly constrained mainly by the  $\mu \rightarrow e\gamma$  constraint. As a consequence, the lepton flavor violating Higgs boson decays  $h \rightarrow e\mu$  and  $h \rightarrow e\tau$  are strongly suppressed so that the near future experiments such as the ones at the LHC could not observe these decay modes, contrary to the  $h \rightarrow \mu\tau$  mode. Therefore, the non-observation of these decays is one of interesting predictions of this scenario.

## VI. SUMMARY

The anomalous event in  $h \rightarrow \mu\tau$  has been observed by the CMS collaboration. The discrepancy of the muon  $g-2$  is also one of the longstanding issues in the particle physics. These anomalous phenomena may be a hint of physics beyond the Standard Model. At glance, these anomalies are not related each others. However, we have found that the both anomalies are related and accommodated by the  $\mu - \tau$  flavor violating Yukawa interactions in a general two Higgs doublet model, and hence this motivates further studies to see whether there are any interesting predictions and indications in the scenario. We have identified the parameter space where the CMS excess in  $h \rightarrow \mu\tau$  and the muon  $g-2$  anomaly are both explained, and especially we have studied  $\tau$ - and  $\mu$ - physics in this interesting parameter space.

One of the interesting processes in the presence of the  $\mu - \tau$  flavor violation is  $\tau \rightarrow \mu\gamma$ . The  $\mu - \tau$  flavor violation suggested by the CMS excess in  $h \rightarrow \mu\tau$  and the muon  $g-2$  anomaly induces the large branching ratio, and it can be as large as  $10^{-9}$  which is within the reach of the future experiment at the SuperKEKB. The imaginary parts of the  $\mu - \tau$  flavor violating Yukawa couplings also induce the extra contributions to the muon EDM, which may be also within the planning future experiments. The necessary  $\mu - \tau$  flavor violation also generates the correction to  $\tau \rightarrow \mu\nu\bar{\nu}$  decay and also induces a violation of lepton universality between  $\tau \rightarrow \mu\nu\bar{\nu}$  and  $\tau \rightarrow e\nu\bar{\nu}$ . The

improvement of their precisions would be interesting. The tree-level  $\tau$  decays such as  $\tau^- \rightarrow \mu^- l^+ l^-$  ( $l = e, \mu$ ) and  $\tau \rightarrow \mu \eta$  are also interesting because the extra Yukawa couplings  $\rho_e^{ee(\mu\mu)}$  and  $\rho_d^{ss}$  could also induce the observable effects. On the other hand, we have found that the  $e-\mu$  and  $e-\tau$  flavor violating Yukawa couplings are severely constrained by mainly  $\mu \rightarrow e\gamma$  process. Because of these constraints, phenomena such as  $\tau \rightarrow e\gamma$ ,  $\tau^- \rightarrow e^- l^+ l^-$  ( $l = e, \mu$ ),  $e^- \mu^+ e^-$ ,  $\mu^- e^+ \mu^-$  and extra contributions to the electron  $g-2$  would not be accessible in the near future experiments. Although there are many unknown Yukawa couplings in a general 2HDM, there are many interesting indications to  $\tau$ - and  $\mu$ -physics.

We have also commented on an implication to Higgs physics. Contrary to the  $\mu-\tau$  flavor violation suggested by the CMS result, the  $e-\mu$  and  $e-\tau$  flavor violations in the Higgs coupling are strongly limited. Therefore, the observation of  $h \rightarrow \mu\tau$  and non-observation of  $h \rightarrow e\mu$  and  $h \rightarrow e\tau$  would be the important implication of the scenario.

We summarize our findings in Table I. If the CMS excess in  $h \rightarrow \mu\tau$  is justified in coming LHC run, these phenomena in  $\tau$ - and  $\mu$ - physics would be key to reveal the physics beyond the Standard Model.

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Process	typical value	observability
muon g-2	$\delta a_\mu = (2.6 \pm 0.8) \times 10^{-9}$	(input)
$\tau \rightarrow \mu\gamma$	$\text{BR} \leq 10^{-9}$	○
$\tau \rightarrow e\gamma$	small	×
$\tau \rightarrow \mu l^+ l^-$ ( $l = e, \mu$ )	depends on $\rho_e^{\mu\mu}$ and $\rho_e^{ee}$	(○)
$\tau^- \rightarrow e^- l^+ l^-, e^- \mu^+ e^-, \mu^- e^+ \mu^-$	small	×
$\tau \rightarrow \mu\eta$	depends on $\rho_d^{ss}$	(○)
$\tau \rightarrow \mu\nu\bar{\nu}$	$\delta \leq 10^{-3}$ , lepton non-universality	△
$\tau \rightarrow e\nu\bar{\nu}$	small, lepton non-universality	△
$\mu \rightarrow e\gamma$	depends on $\rho_e^{\tau e(e\tau)}$ and $\rho_e^{\mu e(e\mu)}$	(○)
$\mu - e$ conversion	depends on $\rho_e^{\mu e(e\mu)}$ and $\rho_{d,u}^{ij}$	(○)
$\mu \rightarrow 3e$	$\text{BR} \leq 10^{-13}$	(○)
muon EDM	$ \delta d_\mu  \leq 10^{-22} e \cdot \text{cm}$	(○)
electron g-2	small	×
LFV Higgs decay mode	BR	
$h \rightarrow \mu\tau$	$\text{BR} = (0.84^{+0.39}_{-0.37})\%$	(input)
$h \rightarrow e\tau$	small	×
$h \rightarrow e\mu$	small	×

TABLE I: Observabilities in various processes are summarized. If there is an observability in the planning experiments without introducing unknown Yukawa couplings  $\rho_f$  other than  $\rho_e^{\mu\tau}$  ( $\tau\mu$ ), the circle mark “○” is shown. If there is an observability, but it depends on unknown Yukawa couplings (other than  $\rho_e^{\mu\tau}$  ( $\tau\mu$ )), “(○)” is indicated. If there is an observability when the (currently unknown) experimental improvement is achieved, the triangle mark “△” is shown. If the event rate is expected to be too small to be observed, “×” is shown.

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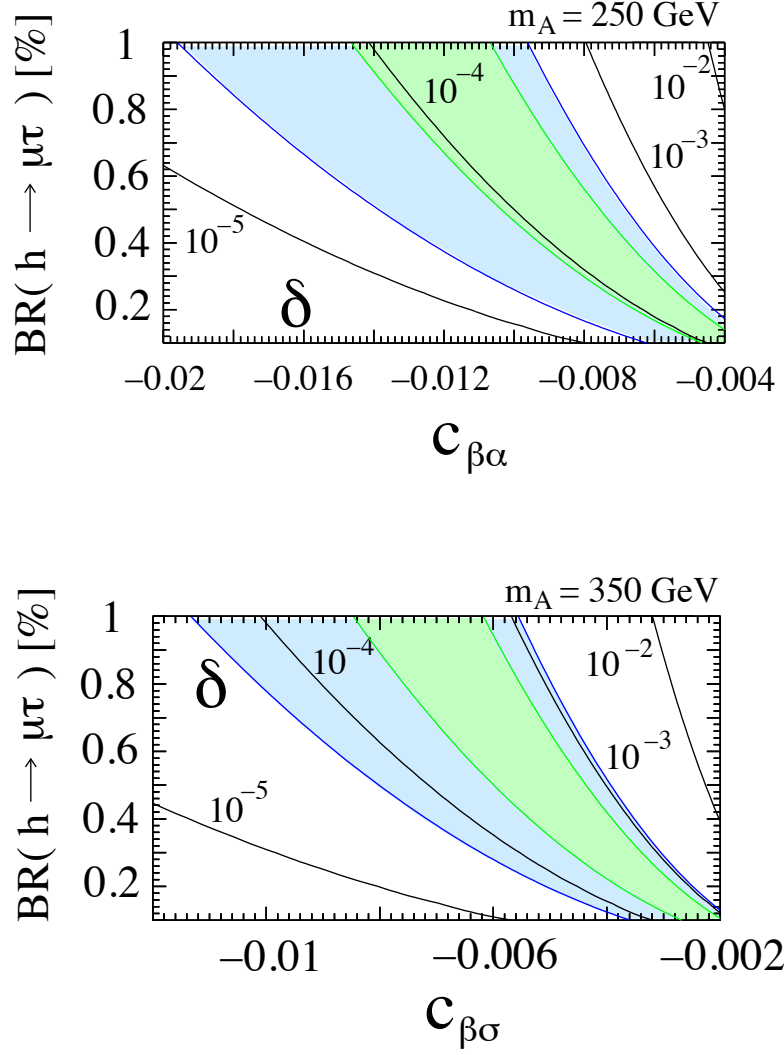


FIG. 10: Numerical result for a correction to  $\tau \rightarrow \mu\nu\bar{\nu}$ ,  $\delta$  given in Eq. (39) as a function of  $c_{\beta\alpha}$  and  $\text{BR}(h \rightarrow \mu\tau)$  in the same parameter set of Fig. 2. Black solid lines for  $\delta = 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$  (from left to right) are shown for  $m_A = 250 \text{ GeV}$  (upper figure) and  $m_A = 350 \text{ GeV}$  (lower figure). Here we also show the region where the muon g-2 anomaly is explained within  $1\sigma$  (light green) and  $2\sigma$  (light blue). Here we have assumed that the extra Yukawa couplings  $\rho_f$  other than  $\rho_e^{\mu\tau(\tau\mu)}$  are negligible.

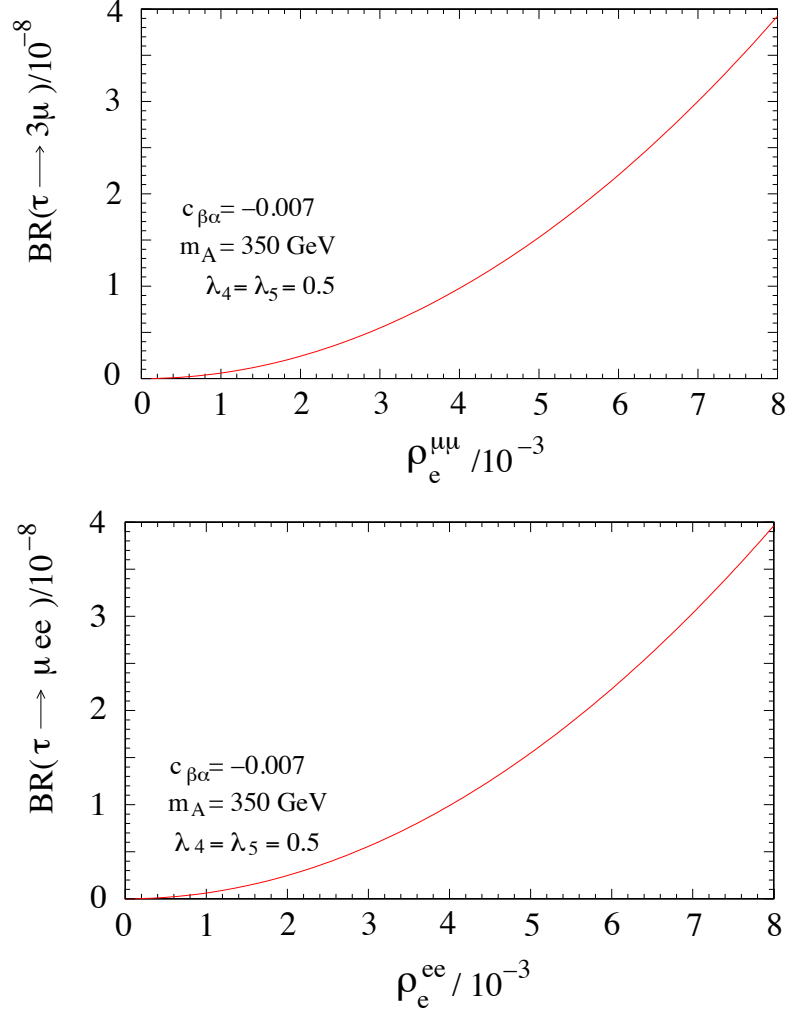


FIG. 11:  $\text{BR}(\tau \rightarrow 3\mu)$  (above) and  $\text{BR}(\tau \rightarrow \mu ee)$  (below) as a function of  $\rho_e^{ll}$  ( $l = \mu$  for  $\tau \rightarrow 3\mu$  and  $l = e$  for  $\tau \rightarrow \mu ee$ ). Here we have assumed that  $c_{\beta\alpha} = -0.007$ ,  $m_A = 350 \text{ GeV}$  with  $\lambda_4 = \lambda_5 = 0.5$  and  $\text{BR}(h \rightarrow \mu\tau) = 0.84\%$  with  $\rho_e^{\mu\tau} = -\rho_e^{\tau\mu}$ .

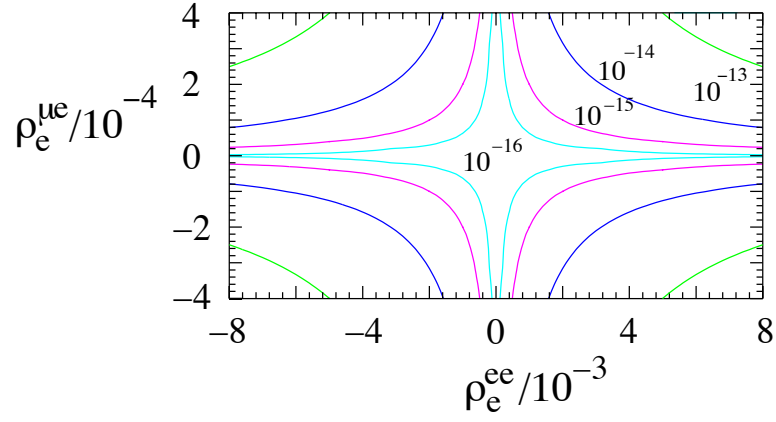


FIG. 12:  $\text{BR}(\mu \rightarrow 3e)$  as a function of  $\rho_e^{ee}$  and  $\rho_e^{\mu e}$ . Here we have assumed that  $\rho_e^{\mu e} = \rho_e^{e\mu}$ ,  $c_{\beta\alpha} = -0.007$  and  $m_A = 350$  GeV with  $\lambda_4 = \lambda_5 = 0.5$ .